INTERNATIONAL BACCALAUREATE

Mathematics: applications and interpretation

MAI

EXERCISES [MAI 5.2-5.3]

BASIC DERIVATIVES – TANGENT AND NORMAL

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A. Paper 1 questions (SHORT)

DERIVATIVES OF POWER FUNCTIONS

1. [Maximum mark: 20]

Differentiate the following functions:

Function	Derivative
$y = 7x^3 + 5x^2 + 2x + 3$	
$y = \frac{7}{3}x^3 - \frac{5}{2}x^2 + \frac{1}{3}x + \frac{4}{5}$	
$y = \frac{7x^3}{3} - \frac{5x^2}{2} + \frac{x}{3} + \frac{4}{5}$	
$y = 1 + \frac{2}{x} + \frac{3}{x^2}$	
$y = \frac{1}{3} + \frac{2}{5x} + \frac{3}{7x^2}$	
$y = x^2(1 + \frac{2}{x} + \frac{3}{x^2})$	
$y = \sqrt{x} + \sqrt[3]{x}$	
$y = \sqrt{x^3} + \sqrt[3]{x^2}$	
$y = \frac{1 + x + x^2}{x^2}$	
$y = \frac{3 + 5x + 7x^2}{2x^2}$	

2. [Maximum mark: 6]

Let $f(x) = 5x^2 + 3$

(a) Find $f'(x)$. [2]

(b) Find the gradient of the curve $y = f(x)$ at $x = 1$. [1]

(c) Find the coordinates of the point where the gradient is 20. [3]

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3. [Maximum mark: 6]

Let $f(x) = 4\sqrt{x}$

(a) Find $f'(x)$. [2]

(b) Find the gradient of the curve $y = f(x)$ at $x = 1$. [1]

(c) Find the coordinates of the point where the gradient is 1. [3]

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4. [Maximum mark: 4]

Let $f(x) = x^3 - 2x^2 - 1$.

(a) Find $f'(x)$ [2]

(b) Find the gradient of the curve of $f(x)$ at the point $(2, -1)$. [2]

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5. [Maximum mark: 6]

Given the function $f(x) = x^2 - 3bx + (c+2)$, determine the values of b and c such that $f(1) = 0$ and $f'(3) = 0$.

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6. [Maximum mark: 7]

Given the following values at $x=1$

x	$f(x)$	$g(x)$	$f'(x)$	$g'(x)$
1	2	3	4	5

(a) Find the value of each function below at $x=1$.

(i) $y=f(x)+g(x)$

(ii) $y=2f(x)+3g(x)$

(iii) $y=f(x)+5x^2$ [3]

(b) Calculate the derivatives of the following functions at $x=1$

(i) $y=f(x)+g(x)$

(ii) $y=2f(x)+3g(x)$

(iii) $y=f(x)+5x^2$ [4]

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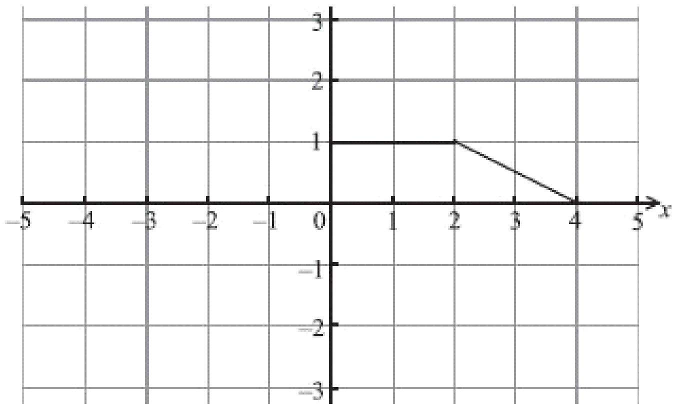
7. [Maximum mark: 4]

The graph of the function $y = f(x)$, $0 \le x \le 4$, is shown below.



(a) Write down the value of (i) $f(1)$ (ii) $f(3)$

(b) Write down the value of (i) $f'(1)$ (ii) $f'(3)$

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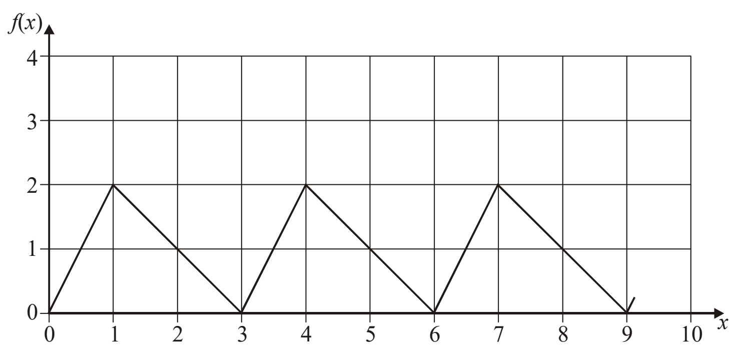
8. [Maximum mark: 6]

Part of the graph of the periodic function f is shown below. The domain of f is $0 \le x \le 15$ and the period is 3.



(a) Find (i) $f(2)$ (ii) $f'(6.5)$ (iii) $f'(14)$

(b) How many solutions are there to the equation $f(x) = 1$ over the given domain?

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TANGENT LINE – NORMAL LINE

9. [Maximum mark: 8]

Let $f(x) = 2x^2 - 12x + 10$.

(a) Find $f'(x)$. [1]

(b) Find the equations of the tangent line and the normal line at $x = 2$. [4]

(c) Find the equations of the tangent line and the normal line at $x = 3$. [3]

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10. [Maximum mark: 9]

Let $f(x) = 2x^2 - 12x + 10$.

(a) Find $f'(x)$. [1]

(b) The line L_1 with equation $y = 4x - 22$ is tangent to the curve.

(i) Write down the gradient of the line L_1.

(ii) Find the coordinates of the point where the line L_1 touches the curve. [3]

(c) The line L_2 is tangent to the curve and parallel to the line $y = 8x + 3$.

(i) Write down the gradient of the line L_2.

(ii) Find the coordinates of the point where the line L_2 touches the curve.

(iii) Find the equation of L_2 in the form $y = mx + c$. [5]

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11. [Maximum mark: 6]

Find the equation of the tangent line and the equation of the normal to the curve with equation $y = x^3 + 1$ at the point (1,2).

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12. [Maximum mark: 6]

Consider the function $f(x) = 4x^3 + 2x$. Find the equation of the normal to the curve of f at the point where $x = 1$.

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13. [Maximum mark: 4]

Find the coordinates of the point on the graph of $y = x^2 - x$ at which the tangent is parallel to the line $y = 5x$.

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14. [Maximum mark: 6]

Let $f(x) = kx^4$. The point $\mathrm{P}(1, k)$ lies on the curve of f. At P, the normal to the curve is parallel to $y = -\frac{1}{8}x$. Find the value of k.

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15. [Maximum mark: 6]

Consider the function $f : x \mapsto 3x^2 - 5x + k$.

The equation of the tangent to the graph of f at $x = p$ is $y = 7x - 9$.

(a) Write down $f'(x)$.

(b) Find the value of (i) p; (ii) k.

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16. [Maximum mark: 6]

Consider the curve with equation $f(x) = px^2 + qx$, where p and q are constants.

The point A(1, 3) lies on the curve. The tangent to the curve at A has gradient 8.

Find the value of p and of q.

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17. [Maximum mark: 6]

Consider the tangent to the curve $y = x^3 + 4x^2 + x - 6$.

(a) Find the equation of this tangent at the point where $x = -1$.

(b) Find the coordinates of the point where this tangent meets the curve again.

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18. [Maximum mark: 6]

The line $y = 16x - 9$ is a tangent to the curve $y = 2x^3 + ax^2 + bx - 9$ at the point (1,7). Find the values of a and b.

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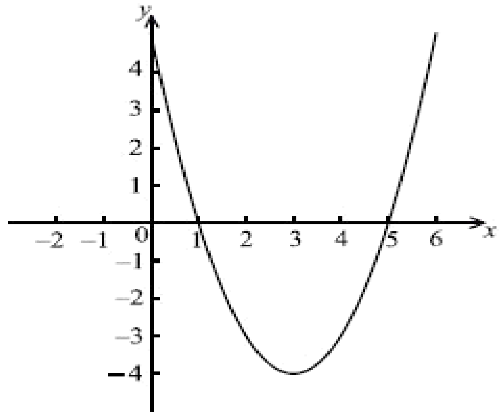
B. Paper 2 questions (LONG)

19. [Maximum mark: 11]

The following diagram shows part of the graph of a quadratic function, with equation in the form $y = (x - p)(x - q)$, where $p, q \in \mathbb{Z}$.



(a) (i) Write down the value of p and of q.

(ii) Write down the equation of the axis of symmetry of the curve. [3]

(b) Find the equation of the function in the form $y = (x - h)^2 + k$, where $h, k \in \mathbb{Z}$. [2]

(c) Find $\frac{\mathrm{d}y}{\mathrm{d}x}$ [3]

(d) Let *T* be the tangent to the curve at the point $(0, 5)$. Find the equation of *T*. [3]

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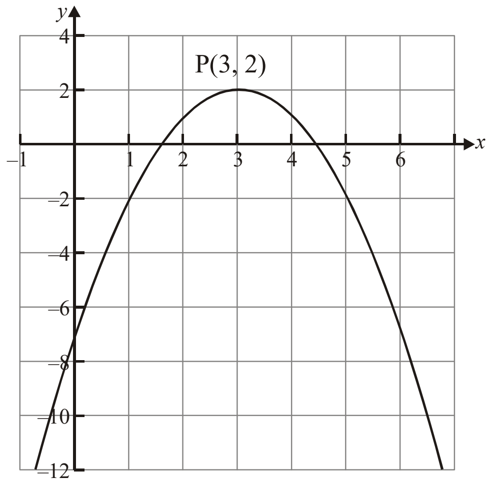
20. [Maximum mark: 13]

The function $f(x)$ is defined as $f(x) = -(x-h)^2 + k$. The diagram below shows part of the graph of $f(x)$. The maximum point on the curve is P (3, 2).



(a) Write down the value of (i) h (ii) k [2]

(b) Show that $f(x)$ can be written as $f(x) = -x^2 + 6x - 7$. [1]

(c) Find $f'(x)$. [2]

The point Q lies on the curve and has coordinates (4, 1). A straight line *L*, through Q, is perpendicular to the tangent at Q.

(d) (i) Find the equation of *L*.

(ii) The line *L* intersects the curve again at R. Find the x-coordinate of R. [8]

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21. [Maximum mark: 11]

The function f is defined by $f : x \mapsto -0.5x^2 + 2x + 2.5$.

(a) Write down (i) $f'(x)$; (ii) $f'(0)$. [2]

(b) Let *N* be the normal to the curve at the point where the graph intercepts the y-axis. Show that the equation of *N* may be written as $y = -0.5x + 2.5$. [3]

Let $g : x \mapsto -0.5x + 2.5$

(c) (i) Find the solutions of $f(x) = g(x)$

(ii) Hence find the coordinates of the other point of intersection of the normal and the curve. [6]

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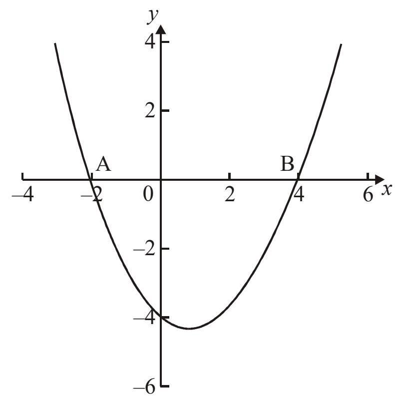
22. [Maximum mark: 15]

The equation of a curve may be written in the form $y = a(x - p)(x - q)$. The curve intersects the x-axis at A(–2, 0) and B(4, 0). The curve of $y = f(x)$ is shown in the diagram below.



(a) (i) Write down the value of p and of q.

(ii) Given that the point (6, 8) is on the curve, find the value of a.

(iii) Write the equation of the curve in the form $y = ax^2 + bx + c$. [5]

(b) A tangent is drawn to the curve at a point P. The gradient of this tangent is 7. Find the coordinates of P. [4]

(c) The line L passes through B(4, 0), and is normal to the curve at B.

(i) Find the equation of L.

(ii) Find the x-coordinate of the point where L intersects the curve again. [6]

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23. [Maximum mark: 9]

Let $f(x) = x^3 - 3x^2 - 24x + 1$.

(a) Find $f'(x)$ [2]

The tangents to the curve of f at the points P and Q are parallel to the x-axis, where P is to the left of Q.

(b) Calculate the coordinates of P and of Q. [3]

Let N_1 and N_2 be the normals to the curve at P and Q respectively.

(c) Write down the coordinates of the points where

(i) the tangent at P intersects N_2;

(ii) the tangent at Q intersects N_1. [4]

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24. [Maximum mark: 10]

Consider the curve with equation $f(x) = 3x^2$. The point P$(a, 3a^2)$ lies on the curve.

(a) Find the gradient to the curve at $x = a$. [2]

(b) Show that the equation of the tangent line to the curve at point P$(a, 3a^2)$ has equation $y = 6ax - 3a^2$. [3]

(c) Given that the tangent line passes through the point $A(0, -3)$ find the possible values of a. [3]

(d) **Hence**, find the equations of the tangent lines **passing through** $A(0, -3)$. [2]

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25. [Maximum mark: 10]

Consider the curve with equation $f(x) = 2x^3$.

(a) Find the equation of the tangent line to the curve at $x = 1$. [3]

(b) Find in terms of a the equation of the tangent line to the curve at $x = a$. [3]

(c) **Hence**, find the equation of the tangent line **passing through** the point $A(0, 4)$. [4]

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